High Energy Facilities
Advanced Projects

RHIC-13

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Beam Life Time in the Presence of Beam Blow Up

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1. Introduction

The life time of the beam particles in the accelerators is determined by (1) the beam dynamics in the presence of nonlinear magnetic elements, (2) the interaction between the beam particles and the gas molecules in the vacuum tube, (3) the interaction between the particles in the two colliding beams and (4) the beam-blow up rate in the presence of collective effect, intrabeam scattering[1,2,3] etc.

This note is to evaluate the beam life time of the relativistic heavy ion collider, where the intrabeam Coulomb scattering is an important factor in the design study.

2. Beam life time due to reaction between two colliding beams

When the beam life time is determined mainly by beam-beam reaction, the reactin rate can be expressed as

$$\lambda = 6 \ \mathscr{L} \sigma_{bb}/BN \tag{1}$$

where $\mathscr{L}$, σ_{bb}, B and N_B are the luminosity, the reaction cross section, the number of bunches in the accelerator and the number of particles per bunch. The factor 6 in Eq. (1) comes from 6 interaction points in RHIC. The total cross section σ_{bb} consists of nuclear reaction, Coulomb reaction and Bremsstrahlung with pair production and subsequent e^- capture (4) causing the beam to be lost. Table 1 shows the beam life time of RHIC due to various reaction mechanism.

3. Effect of beam blow up on the life time

The reaction rate λ defined in Eq. (1) depends on the luminosity and the number of particles in the beam. After some simple algebra, we obtain

$$\lambda(t) = \frac{9\gamma N_B \sigma_{bb} \ f_{rev}}{\beta^* \varepsilon_N} \tag{2}$$

Table 1. Initial Reaction Rate $\lambda = -I^{-1}\, dI/dt$ and Total Half Life of Particle Beams for Head-on Collisions

	Beam-gas nuclear reaction λ_1	Beam-beam nuclear reaction λ_2		Beam-beam Coulomb dissociation λ_3	Beam-beam[4] Bremsstrahlung electron pair production λ_4	Initial Half Life
Beam	@ 10^{-10} Torr	A on A	p on A	A on A	A on A	A on A
	$\times 10^{-3}$/h	$\times 10^{-3}$/h	$\times 10^{-3}$/h	$\times 10^{-3}$/h	$\times 10^{-3}$/h	h
p	0.15	1.6	1.6	--	--	396
d	0.19	8.7	3.0	--	--	78
C	0.36	5.2	16.9	--	--	125
S	0.55	4.6	27.9	--	--	135
Cu	0.76	4.6	38.8	0.52	0.12	116
I	1.08	3.9	55.1	13.2	4.6	30
Au	1.37	2.1	69.3	16.0	31.6	14

where γ, N_B, ε_N, f_{rev}, σ_{bb} and β^* are the Lorentz relativistic factor, number of particles per bunch, the normalized emittance, the revolution frequency, total cross section between particles in two beams and the betatron amplitude function at the interaction point respectively. Since N_B and ε_N are both function of time, we obtain

$$\frac{1}{\lambda}\frac{d\lambda}{dt} = \frac{1}{N_B}\frac{dN_B}{dt} - \frac{1}{\varepsilon_N}\frac{d\varepsilon_N}{dt} = -\lambda - \mu(t) \quad . \tag{3}$$

The equation for the reaction rate is indeed nonlinear. When the emittance blow up rate $\mu(t)$ is taken to be a CONSTANT, Eq. (3) can easily be solved to give,

$$\lambda = \lambda_o\, e^{-\mu t}/[1 + \frac{\lambda_o}{\mu}(1 - e^{-\mu t})] \tag{4}$$

where λ_o is the initial reaction rate. When the beam blow-up rate is faster then the reaction rate, i.e., $\mu >> \lambda_o$, we obtain

$$\lambda \simeq \lambda_o\, e^{-\mu t} \quad . \tag{5}$$

On the other hand, when the beam does not blow up, i.e., $\mu = 0$, we have $\lambda \simeq \lambda_o$. In the heavy ion collider, we encounter $\mu >> \lambda_o$ (3).

From Eq. (4), we obtain then

$$\begin{aligned} I(t)/I_o &= (1 + \frac{\lambda_o}{\mu}(1 - e^{-\mu t}))^{-1} \\ &\simeq \exp\left(\frac{\lambda_o}{\mu}(1 - e^{-\mu t})\right) . \end{aligned} \tag{6}$$

Figure 1 shows $\lambda(t)$ and $N(t)$ as a function of time for Au ion [eqs. (4) and (6)] with parameters $\mu = 0.1/\text{hr}$, $\lambda_o = .0495/\text{hr}$. The number of particles $N(t)$ is to be compared with the constant λ_o decay law of $e^{-\lambda_o t}$.

4. Luminosity

The luminosity depends on the beam size as well as the intensity of the beam,

$$\mathscr{L}(t) = 6\, \frac{\gamma N_B^2 B\, f_{rev}}{4\varepsilon_N \beta^*} \quad . \tag{7}$$

Thus,

$$\frac{1}{\mathscr{L}}\frac{d\mathscr{L}(t)}{dt} = -2\lambda - \mu \tag{8}$$

or,

$$\mathscr{L}(t)/\mathscr{L}_o = e^{-\mu t}/(1 + \frac{\lambda_o}{\mu}(1 - e^{-\mu t}))^2 . \tag{9}$$

Figure 1 shows the luminosity as a function of time for Au ion. The average luminosity for a T hours run becomes

$$<\mathscr{L}>/\mathscr{L}_o = \frac{1}{\mu T}(1 - e^{-\mu T})/(1 + \frac{\lambda_o}{\mu}(1 - e^{-\mu T})) . \tag{10}$$

The average luminosity for T = 10 hours operation of Au on Au becomes

$$<\mathscr{L}>/\mathscr{L}_o = 0.48 \; (\mu = 0.1/\text{hr}, \; \lambda_o = 0.0495/\text{hr}) .$$

For the most other ions, the Coulomb dissociation and pair production cross sections are rather small[4). The reaction rate is irrelevant in determining the average luminosity. The average luminosity is given by (for $\mu = 0.1/\text{hr}$)

$$<\mathscr{L}>/\mathscr{L}_o = (1 - e^{-\mu T})/\mu T \simeq .63$$

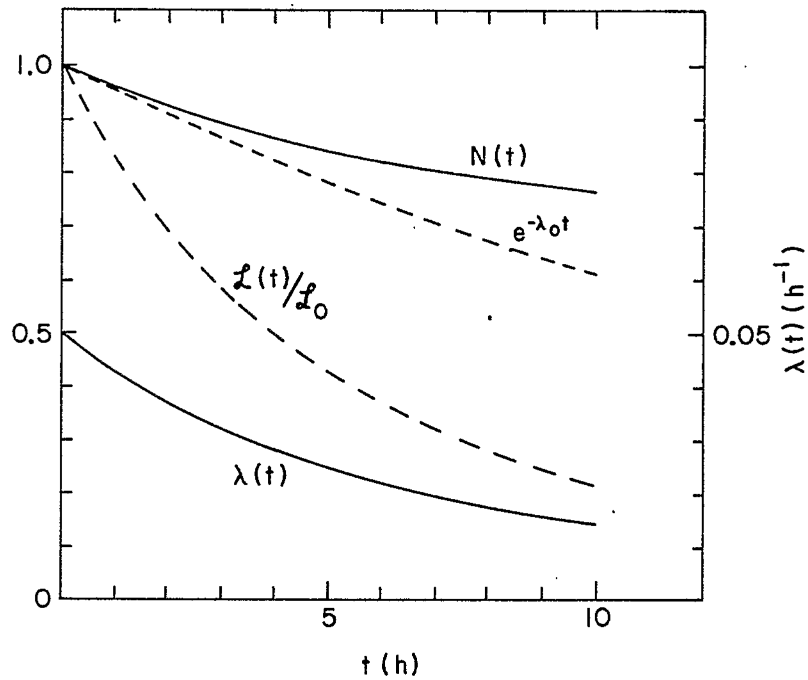

Fig. 1. The beam depletion rate, $\lambda(t)$, the number of particles, $N(t)/N_o$, and the luminosity $\mathscr{L}/\mathscr{L}_o$ are plotted as a function of time t. μ = 0.1/hr, λ_o = .0495/hr for Au ion is used in this calculation.

APPENDIX

In general, the beam emittance blow up rate $\mu(t)$ is not a constant. Intrabeam scattering calculation shows that[5)]

$$\varepsilon_N(t) \simeq \varepsilon_N^o + \alpha\sqrt{t}\ . \qquad (A.1)$$

We obtain therefore

$$\mu(t) = \frac{1}{\varepsilon_N}\frac{d\varepsilon_N}{dt} = \frac{\frac{1}{2}\alpha t^{-\frac{1}{2}}}{\varepsilon_N^o + \alpha\sqrt{t}}\ . \qquad (A.2)$$

The solution of Eq. (3) becomes

$$\lambda(t) = e^{-\int_o^t \mu(t)dt}\left(\frac{1}{\lambda_o} + \int_o^t e^{-\int^{t'}\mu dt''} dt'\right)^{-1}$$

$$= \lambda_o \Big/ (1 + x)\ \left\{1 + 2\ \lambda_o\left(\frac{\varepsilon_N^o}{\alpha}\right)^2 \left(x - \ln(1 + x)\right)\right\} \qquad (A.3)$$

where

$$x(t) = \frac{\alpha\sqrt{t}}{\varepsilon^o_N}\ . \qquad (A.4)$$

Whence the intensity and the luminosity become

$$I = I_o\ e^{-\int_o^t \lambda(t')dt'} \qquad (A.5)$$

$$\mathscr{L} = \mathscr{L}_o\ e^{-2\int_o^t \lambda(t')dt'} \Big/ (1 + x(t))\ . \qquad (A.6)$$

References

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